Symbolic Arithmetic of Physical Constants

Foundations of a New Branch:
Complexity Hierarchies for Dimensionless Constants

Judicael Brindel

Independent Researcher, Cotonou, Benin

ORCID: 0009-0007-4590-9874

March 2026

Version 1.0 — Foundational Proposal

Abstract

We propose the foundations of a new branch of arithmetic — the *Symbolic Arithmetic of Physical Constants* — which asks whether the dimensionless constants of physics occupy specific, identifiable positions in a hierarchy of symbolic complexity, or whether they are distributed arbitrarily among the reals.

This question is distinct from Kolmogorov complexity (which concerns arbitrary strings) and from empirical formula-hunting (which lacks statistical rigour). We define a formal complexity metric S over a symbolic grammar $\mathcal{G}$ of physical and mathematical constants, establish a five-level complexity hierarchy for real numbers, and place known constants within it.

The first empirical result of this branch — a negative result at grammar depth $d = 3$ showing no statistically significant enhanced compressibility of physical constants over random numbers (Mann-Whitney $p = 0.2754$) — has already been established [1]. The open-source tool SymSearch [2] provides a reproducible experimental platform for all future investigations in this branch.

All foundational claims are clearly marked by status.

Contents

1 Motivation: An Unanswered Question in Arithmetic **3**

2 What Exists and What Is Missing **3**

2.1 What exists 3

2.2 What is missing 3

3 The Symbolic Grammar $\mathcal{G}$ **3**

4 The Complexity Metric S **4**

5 The Complexity Hierarchy **4**

6 A Concrete Example: The CMB Temperature **4**

7 The First Empirical Result 5

8 SymSearch as Foundational Tool 5

9 The Foundational Conjecture 5

10 Open Problems 6

11 Summary 7

1 Motivation: An Unanswered Question in Arithmetic

The fundamental constants of physics — the fine-structure constant $\alpha^{-1} \approx 137.036$, the proton-to-electron mass ratio $m_p/m_e \approx 1836.15$, the weak mixing angle $\sin^2\theta_W \approx 0.231$ — are measured with extraordinary precision. But their *arithmetic nature* remains unknown.

Are these numbers special — compressible, structured, related to classical mathematical constants in some deep way? Or are they arbitrary real numbers, no more structured than a random draw from $[0, \infty)$?

This question is not physics. It is *arithmetic* — the study of the properties of numbers. Kolmogorov [7] and Chaitin [8] developed a general theory of algorithmic complexity for arbitrary strings. But they did not ask this question specifically for the constants of nature.

We propose that this question deserves its own branch.

2 What Exists and What Is Missing

2.1 What exists

Kolmogorov complexity (1965). The complexity $K(x)$ of a string x is the length of the shortest program that generates x. Random strings have maximal complexity. Structured strings are compressible.

Chaitin's Ω (1975). Some real numbers are algorithmically random — their binary expansion contains no compressible patterns. Most reals are of this type.

Empirical formula searches. From Balmer (1885) to Wyler (1969) to Chekanov (2025) [9], numerous authors have proposed symbolic formulas for specific constants. But these searches are typically post-hoc, lack control groups, and provide no statistical test of significance.

2.2 What is missing

No branch of arithmetic has systematically asked: *what is the symbolic complexity structure of the set of dimensionless physical constants as a whole?*

This is the gap the Symbolic Arithmetic of Physical Constants fills.

3 The Symbolic Grammar $\mathcal{G}$

Definition 1 (Symbolic grammar $\mathcal{G}$)**.** *The grammar $\mathcal{G}$ is a context-free grammar over free syntax trees with:*

- ***Terminal nodes (leaves):*** *mathematical constants* $\{\pi, e, \phi, \sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \ln 2, \zeta(3), \gamma_{\mathrm{EM}}\}$, *physical constants (68 dimensionless values from PDG 2024 and CODATA 2018), and rational atoms a/b with $a \in [-50, 50] \setminus \{0\}$, $b \in [1, 20]$.*
- ***Internal nodes:*** *unary operators* $\{\mathrm{neg}, \mathrm{inv}, \sqrt{\cdot}, \log, \exp, (\cdot)^2, (\cdot)^3\}$ *and binary operators* $\{+, -, \times, \div, \hat{}\}$.
- ***Depth bound:*** $d \le d_{\max}$, *configurable.*

The grammar is fully specified in [2] and implemented in the open-source tool SymSearch.

4 The Complexity Metric S

Definition 2 (Symbolic complexity S)**.** *For an expression e represented as a syntax tree:*

$$S(e) = W_1 N_{\text{op}} + W_2 \,\text{depth}(e) + W_3 N_{\text{const}} + W_4 \sum_c \log_2(1 + |c|)$$

where N_{op} is the number of operations, $\text{depth}(e)$ *is the maximum tree depth, N_{const} is the number of distinct constants used, and the sum runs over all integer coefficients c. Weights $(W_1, W_2, W_3, W_4) = (1.0, 0.8, 0.5, 0.3)$ are fixed prior to any computation.*

The symbolic complexity of a real number x *under grammar $\mathcal{G}$ at depth d and tolerance σ is:*

$$S_{\min}(x, \mathcal{G}, d, \sigma) = \min\{S(e) : e \in \mathcal{G}_d,\ |\text{eval}(e) - x| \le \sigma\}$$

If no matching expression exists, $S_{\min} = S_\infty = 1000$.

Remark 1. *$S_{\min}$ depends on $\mathcal{G}$, d, and σ. This is a feature, not a bug — it makes the complexity measurement explicit and reproducible, unlike Kolmogorov complexity which is uncomputable.*

5 The Complexity Hierarchy

Definition 3 (Symbolic complexity hierarchy)**.** *We define five levels of symbolic complexity for real numbers:*

Level	***Name***	$S_{\min}$ ***range***	***Examples***
0	*Trivial*	$S = 0$	$0, 1, -1$
1	*Elementary*	$S \le 1.5$	$\pi, e, \phi, \alpha^{-1}, m_p/m_e$
2	*Structured*	$1.5 < S \le 4$	T_{CMB} *(proposed)*, α_s
3	*Complex*	$4 < S \le 10$	$\sin^2\theta_W$, *quark mass ratios*
4	*Opaque*	$S = S_\infty$	*Unknown at current depth*

A number is symbolically random *if $S_{\min} \to S_\infty$ as $d \to \infty$.*

6 A Concrete Example: The CMB Temperature

The following observation motivates Level 2 of the hierarchy.

Proposition 1 (CMB temperature — proposed derivation)**.** *The following expression involving the Landauer limit energy $E_L = k_B T \ln 2$, the Heisenberg uncertainty principle, and atomic constants yields a value consistent with the observed CMB temperature $T_{\text{CMB}} = 2.72548 \pm 0.00057$ K:*

$$T_{\text{CMB}} \approx f(\hbar, c, k_B, m_e, \alpha)$$

The full derivation is given in [3]. This expression has symbolic complexity $S \approx 3$ under $\mathcal{G}$ — placing T_{CMB} at Level 2.

[Sketched] *(the derivation is proposed and not yet independently verified; included here as a concrete example of Level 2 structure)*

Remark 2. *Whether T_{CMB} is truly derivable from atomic constants or whether the numerical agreement is coincidental is precisely the kind of question the Symbolic Arithmetic of Physical Constants is designed to answer rigorously.*

7 The First Empirical Result

Theorem 1 (Negative result at depth $d = 3$)**.** *Under grammar $\mathcal{G}$ with depth $d = 3$, $N = 5{,}000{,}000$ random expressions, and seed $= 42$, a Mann-Whitney U test comparing $S_{\min}$ distributions between 15 dimensionless physical constants and 40 random control numbers yields $p = 0.2754$, failing to reject the null hypothesis at the 0.01 significance level.*

[Established] *(full protocol and code in [1])*

Remark 3. *This negative result is informative. It establishes that if physical constants have enhanced symbolic compressibility, this structure is not detectable at depth $d = 3$ with grammar $\mathcal{G}$. The signal, if it exists, lies at depth $d \geq 4$.*

8 SymSearch as Foundational Tool

The Symbolic Arithmetic of Physical Constants requires a reproducible experimental platform. SymSearch [2] provides this.

Proposition 2 (SymSearch capabilities)**.** *SymSearch implements:*

(i) Forward search: given x and σ, find all expressions $e \in \mathcal{G}_d$ with $|\text{eval}(e) - x| \leq \sigma$, ranked by S.

(ii) Inverse search: given a formula string, identify which known constants it approximates.

(iii) Automatic coincidence catalogue: rare results ($p < 10^{-4}$) are logged automatically.

(iv) Configurable depth $d \in \{1, 2, 3, 4, 5\}$ and sample size N.

All results are deterministic given a fixed seed. Code is open source under MIT licence at `https://github.com/judicaelbrindel/symsearch`. ***[Established]***

9 The Foundational Conjecture

Before listing open problems, we state the central conjecture that motivates the entire branch.

Definition 4 (Symbolically non-random number)**.** *A real number x is* symbolically non-random *under grammar $\mathcal{G}$ if:*

$$S_{\min}(x, \mathcal{G}, d, \sigma) < S_\infty \quad \textit{for some finite depth } d$$

That is, there exists at least one expression in $\mathcal{G}$ that matches x within tolerance σ.

Conjecture 1 (Foundational Conjecture of Symbolic Arithmetic)**.** ***For every dimensionless physical constant c of the Standard Model, there exists a finite depth $d^*(c)$ and a minimal expression $e^* \in \mathcal{G}_{d^*(c)}$ such that:***

$$S(e^*) = S_{\min}(c, \mathcal{G}, d^*(c), \sigma_c)$$

where σ_c is the experimental uncertainty of c.

In other words: the fundamental constants of physics are symbolically non-random. They are not arbitrary real numbers. Each admits a minimal symbolic expression — a most compact mathematical description — waiting to be found.

[Open] *(this is the central open question of the branch)*

Remark 4 (Relation to Chaitin randomness)**.** *Chaitin [8] showed that most real numbers are algorithmically random — no program shorter than the number itself generates them. The Foundational Conjecture claims that the physical constants are* not *of this type.*

If the conjecture is false — if some physical constants are truly Chaitin-random — then no symbolic expression will ever match them at any depth. SymSearch would diverge to S_∞ as $d \to \infty$. This would itself be a profound result: some constants of nature are arithmetically irreducible.

If the conjecture is true, each constant has a depth $d^*(c)$ at which it becomes compressible. Finding these depths — and the expressions that achieve them — is the programme of the Symbolic Arithmetic of Physical Constants.

Remark 5 (Why this is not yet an axiom)**.** *The Foundational Conjecture cannot be elevated to an axiom at this stage because we do not yet know whether physical constants are symbolically non-random. The negative result at $d = 3$ [1] does not decide the question. It constrains it: if the signal exists, it lies at $d \geq 4$.*

When — and if — a statistically significant positive result is established at some depth d^*, the conjecture becomes a theorem, and its statement can be elevated to an axiom of the branch.

10 Open Problems

These are the questions the Symbolic Arithmetic of Physical Constants leaves open — for future researchers, for mathematicians with deeper tools, for anyone who picks up this programme.

1. **Signal at deeper depth. [Open]**
   Does a statistically significant difference in $S_{\min}$ between physical constants and random numbers emerge at $d = 4$ or $d = 5$? The negative result at $d = 3$ constrains but does not close this question.

2. **Nature of α^{-1} and m_p/m_e. [Open]**
   These constants produced no match at $d = 3$ (sigma too tight). Are they Level 1 constants whose simplicity is trivial (they are terminal atoms in $\mathcal{G}$) or do they have deeper structure at higher depth?

3. **Symbolic randomness criterion. [Open]**
   Define formally when a physical constant is *symbolically random* — i.e. $S_{\min} \to S_\infty$ as $d \to \infty$ for any grammar $\mathcal{G}$. Is this equivalent to Kolmogorov randomness?

4. **Independence of physical constants. [Open]**
   Are the dimensionless constants of the Standard Model arithmetically independent — or do some satisfy non-trivial algebraic relations detectable at higher depth?

5. **Grammar extension. [Open]**
   What is the minimal grammar extension needed to match α^{-1} and m_p/m_e within

experimental precision? Does this extension reveal structure in the constants or merely reflect the richness of the grammar?

6. **Connection to Kolmogorov complexity.** **[Open]**
Is $S_{\min}(x, \mathcal{G}, d, \sigma)$ a computable approximation to $K(x)$? Under what conditions do the two measures converge?

11 Summary

Element	Status
Grammar $\mathcal{G}$ defined	**[Established]**
Complexity metric S defined	**[Established]**
Five-level hierarchy	**[Established]**
First empirical result ($d = 3$, negative)	**[Established]**
SymSearch open-source tool	**[Established]**
CMB example at Level 2	**[Sketched]**
Signal at $d \geq 4$	**[Open]**
Symbolic randomness criterion	**[Open]**
Independence of SM constants	**[Open]**

The central proposal:

> *The dimensionless constants of physics are real numbers. Real numbers have arithmetic properties. The arithmetic properties of the constants of nature have never been systematically studied. This is the Symbolic Arithmetic of Physical Constants. It starts here.*

Acknowledgements

Computational assistance: Claude (Anthropic). Related work: RECT [4], Chaining Force [5], Cosmic Lavoisier [6].

References

[1] J. Brindel, *Algorithmic Compressibility of Dimensionless Physical Constants Under Bounded Symbolic Grammars: A Negative Empirical Study*, Zenodo (2026).

[2] J. Brindel, *SymSearch: Symbolic Search for Physical Constants*, Zenodo (2026), doi:10.5281/zenodo.18805643.

[3] J. Brindel, *A Derivation of the CMB Temperature from Atomic Constants*, Zenodo (2026), doi:10.5281/zenodo.18843119.

[4] J. Brindel, *Relational Emergence Complexity Theory (RECT)*, Zenodo (2026), doi:10.5281/zenodo.18860518.

[5] J. Brindel, *The Chaining Force*, Zenodo (2026), doi:10.5281/zenodo.18863588.

[6] J. Brindel, *The Cosmic Lavoisier Principle*, Zenodo (2026), doi:10.5281/zenodo.18864499.

[7] A. N. Kolmogorov, *Three approaches to the quantitative definition of information*, Problems of Information Transmission **1** (1965) 1–7.

[8] G. J. Chaitin, *A theory of program size formally identical to information theory*, J. ACM **22** (1975) 329–340.

[9] S. V. Chekanov, *A simplicity criterion for empirical formulas involving fundamental constants*, arXiv:2507.00225 (2025).

[10] A. Wyler, *L'espace symétrique du groupe des équations de Maxwell*, C. R. Acad. Sci. Paris **269** (1969) 743–745.